

THAT WHICH IS CLAIMED IS:

1. A method of modeling a surface of an object, comprising the steps of:

- 5 generating from an initial triangulation of the surface, a hierarchy of progressively coarser triangulations of the surface by performing a sequence of edge contractions to the initial triangulation;
- connecting the triangulations in the hierarchy using homeomorphisms; and
- 10 homeomorphically mapping edges of a triangulation in the hierarchy back to the initial triangulation.

2. The method of Claim 1, wherein said mapping step comprises homeomorphically mapping edges of a coarsest triangulation in the hierarchy back to the initial triangulation.

3. The method of Claim 2, wherein said mapping step is followed by the step of:

 converting the mapped coarsest triangulation to a quadrangulation by matching pairs of adjacent triangles in the mapped coarsest triangulation.

4. The method of Claim 3, wherein the pairs of adjacent triangles are matched using a weighting function for edges of the triangles.

5. The method of Claim 3, wherein said converting step comprises: decomposing an isolated triangle that cannot be matched in the mapped coarsest triangulation into three quadrangles; and decomposing a quadrangle derived from a matched pair of adjacent

5 triangles into a mesh of four quadrangles.

6. The method of Claim 3, further comprising the step of fitting a respective grid to each of a plurality of quadrangles in the quadrangulation by decomposing each of the quadrangles into k^2 smaller quadrangles, where k is a positive integer greater than one.

7. A method of modeling a surface of an object, comprising the steps of:

generating from an initial triangulation of the surface, a hierarchy of progressively coarser triangulations of the surface by decimating the initial triangulation using a sequence of edge contractions that are prioritized by an error function that measures a respective error caused by the edge contractions in the sequence;

connecting the triangulations in the hierarchy using homeomorphisms; and

homeomorphically mapping edges of a coarsest triangulation in the hierarchy back to the initial triangulation.

8. The method of Claim 7, wherein said generating step comprises generating a first triangulation from the initial triangulation by contracting a first edge in the initial triangulation and measuring a first error associated with the first edge contraction; and wherein said connecting step comprises generating a first simplicial homeomorphism for the first triangulation.

9. The method of Claim 8, wherein said connecting step comprises generating the first simplicial homeomorphism for the first triangulation by determining a fuzzy rank of a submatrix of a fundamental quadric used by the error function to measure the first error.

10. A method of modeling a surface of an object, comprising the steps

of:

generating from an initial triangulation of the surface, a hierarchy of progressively coarser triangulations of the surface by repeatedly
5 decimating the initial triangulation using a sequence of edge contractions that are prioritized by an error function that measures a respective error caused by each of the edge contractions in the sequence, until a coarsest triangulation having a target density of triangles therein is achieved;

connecting the triangulations in the hierarchy using homeomorphisms;

10 and

homeomorphically mapping edges of the coarsest triangulation in the hierarchy back to the initial triangulation.

11. The method of Claim 10, wherein said connecting step comprises generating a respective simplicial homeomorphism for each of the triangulations in the hierarchy by determining a respective fuzzy rank that is attributable to a corresponding edge contraction in the sequence giving rise
5 to the respective triangulation.

12. The method of Claim 11, wherein said mapping step is followed by the step of:

converting the mapped coarsest triangulation to a quadrangulation by matching pairs of adjacent triangles in the mapped coarsest triangulation.

13. The method of Claim 12, wherein the pairs of adjacent triangles are matched using a weighting function for edges of the triangles.

14. The method of Claim 12, wherein said converting step comprises:
decomposing an isolated triangle that cannot be matched in the
mapped coarsest triangulation into three quadrangles; and
decomposing a quadrangle derived from a matched pair of adjacent
5 triangles into a mesh of four quadrangles.

15. The method of Claim 12, further comprising the step of fitting a
respective grid to a plurality of quadrangles in the quadrangulation by
decomposing each of the plurality of quadrangles into k^2 smaller
quadrangles, where k is a positive integer greater than one.

16. A method of generating a model of an object, comprising the steps
of:

decomposing an initial triangulation of the model into a
quadrangulation of the model defined by a plurality of quadrangular
5 patches that are joined together at patch boundaries by:

generating from the initial triangulation of the model a hierarchy
of progressively coarser triangulations of the model using a
sequence of edge contractions to the initial triangulation;

10 connecting the triangulations in the hierarchy using
homeomorphisms;

homeomorphically mapping edges of a coarsest triangulation in
the hierarchy back to the initial triangulation; and

15 converting the mapped coarsest triangulation to the
quadrangulation by matching pairs of adjacent triangles in the
coarsest triangulation.

17. The method of Claim 16, wherein said generating step comprises generating a first triangulation from the initial triangulation by contracting a first edge in the initial triangulation and measuring a first error associated with the first edge contraction; and wherein said connecting step comprises
5 generating a first simplicial homeomorphism for the first triangulation.

18. The method of Claim 17, wherein said mapping step comprises determining an inverse of a composition of the homeomorphisms.

19. A method of modeling a surface of an object, comprising the steps of:

converting a first triangulation of the surface into a second triangulation of the surface by contracting a first edge in the first triangulation;
5 determining a fuzzy rank associated with the first edge contraction; and determining a simplicial homeomorphism based on the fuzzy rank.

20. A method of modeling a surface of an object, comprising the steps of:

converting an initial triangulation of the surface into a quadrangulation of the surface that is homeomorphic to the triangulation by:

5 generating from the initial triangulation a hierarchy of progressively coarser triangulations of the surface by performing a sequence of edge contractions to the initial triangulation; and mapping edges of a triangulation in the hierarchy back to the initial triangulation.

21. The method of Claim 20, wherein said converting step further comprises converting the mapped triangulation to the quadrangulation by matching pairs of adjacent triangles in the mapped triangulation.

22. The method of Claim 20, wherein said converting step comprises:
determining respective first homeomorphisms associated with each of
the triangulations in the hierarchy;

5 determining a composition of the first homeomorphisms; and
determining an inverse of the composition of the first homeomorphism.

23. A method of generating a model of an object, comprising the steps
of:

5 automatically decomposing a first triangulated surface of the model
into a second quadrangulated surface defined by a plurality of
quadrangular patches that are joined together at patch boundaries, without
manually designating and converting at least a majority of a plurality of
triangles within the first triangulated surface into a plurality of quadrangular
patches;

10 fitting a respective grid to each of the plurality of quadrangular patches;
and

generating a respective NURBS patch over each of the fitted grids
associated with each of the plurality of quadrangular patches.

24. The method of Claim 23, wherein said automatically decomposing
step comprises decimating the first triangulated surface through a
sequence of edge contractions that are prioritized by a quadratic error
measure.

25. The method of Claim 24, wherein the quadratic error measure is
based on a coefficient map that weights planes defined by triangles on the
first triangulated surface by a geometric measure of the triangles.

26. The method of Claim 24, wherein the quadratic error measure is based on a coefficient map that weights planes defined by triangles on the first triangulated surface by their angles or by a geometric measure based on area.

27. The method of Claim 23, wherein the second quadrangulated surface is homeomorphic to the first triangulated surface.

28. A method of generating a three-dimensional model of a three-dimensional object, comprising the steps of:

generating a triangulated model of the object;

5 defining a boundary of a hole in the triangulated model by detecting triangles of the triangulated model that have an edge which is not shared by any other triangles in the triangulated model;

filling the hole with a first triangulated surface; and

improving quality of the first triangulated surface by:

10 refining the first triangulated surface into a second triangulated surface having a higher density of triangles therein relative to the first triangulated surface; and

15 decimating the second triangulated surface into a third triangulated surface having a lower density of triangles therein relative to the second triangulated surface, using an algorithm that favors generation of equilateral triangles when edges of triangles in the second triangulated surface are contracted.

29. The method of Claim 28, wherein said decimating step comprises decimating the second triangulated surface into a third triangulated surface using a modified quadric error function that favors generation of triangles having face angles of about 60 degrees by promoting the generation of isoceles triangles when contracting edges of triangles in the second triangulated surface.

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30. The method of Claim 29, wherein said decimating step comprises decimating the second triangulated surface into a third triangulated surface using a preselected range of triangle densities as a constraint.

31. A method of generating a three-dimensional model of an object, comprising the steps of:

decomposing a triangulated model of the object into a first quadrangulation model defined by a first plurality of quadrangular patches that are joined together at first patch boundaries; and

reconfiguring the first quadrangulation model into a second quadrangulation model defined by a second plurality of quadrangular patches that are joined together at second patch boundaries, by performing a plurality of local transformations on the first plurality of quadrangular patches so that a deficiency associated with the second quadrangulation model is less than a deficiency associated with the first quadrangulation model.

32. A method of generating a three-dimensional model of an object, comprising the steps of:

decomposing a triangulated model of the object into a first quadrangulation model defined by a first plurality of quadrangular patches that are joined together at first patch boundaries; and

reconfiguring the first quadrangulation model into a second quadrangulation model defined by a second plurality of quadrangular patches that are joined together at second patch boundaries, by performing a plurality of local transformations on the first plurality of quadrangular patches so that a potential $\Phi_2(Q)$ associated with the second quadrangulation model is less than a potential $\Phi_1(Q)$ associated with the first quadrangulation model.

33. The method of Claim 32, where $\Phi_n(Q) = K\Delta_n(Q) + F_n(Q)$ and $K \geq 2$, $\Delta_n(Q)$ represents a deficiency of a respective nth quadrangulation and $F_n(Q)$ represents the number of quadrangular patches in the respective nth quadrangulation.

34. A method of generating a three-dimensional model of an object, comprising the steps of:

generating a first quadrangulated surface of a first object that is defined by a plurality of quadrangular patches joined together at patch boundaries that define a first patch boundary mesh;

applying the first quadrangulated surface as a template to a point cloud or triangulated surface representation of a second object that is different from the first object; and

modifying the template by adjusting a shape of the first patch boundary mesh to more closely conform to the surface representation of the second object.

35. The method of Claim 34, further comprising the steps of:
fitting a respective grid to each of the plurality of quadrangular patches in the modified template; and
generating a respective NURBS patch over each of the fitted grids.

36. A computer program product that models a surface of an object and comprises a computer-readable storage medium having computer-readable program code embodied in said medium, said computer-readable program code comprising:

5 computer-readable program code that generates from an initial triangulation of the surface, a hierarchy of progressively coarser triangulations of the surface by performing a sequence of edge contractions to the initial triangulation;

10 computer-readable program code that connects the triangulations in the hierarchy using homeomorphisms; and

computer-readable program code that homeomorphically maps edges of a coarsest triangulation in the hierarchy back to the initial triangulation.

37. The product of Claim 36, further comprising computer-readable program code that converts the mapped coarsest triangulation to a quadrangulation by matching pairs of adjacent triangles in the mapped coarsest triangulation.

38. A computer program product that models a surface of an object and comprises a computer-readable storage medium having computer-readable program code embodied in said medium, said computer-readable program code comprising:

5 computer-readable program code means that generates from an initial triangulation of the surface, a hierarchy of progressively coarser triangulations of the surface by decimating the initial triangulation using a sequence of edge contractions that are prioritized by a quadratic error function that measures a respective error caused by each of the edge
10 contractions in the sequence;

computer-readable program code means that connects the triangulations in the hierarchy using homeomorphisms; and

computer-readable program code means that homeomorphically maps edges of a coarsest triangulation in the hierarchy back to the initial triangulation.

39. The product of Claim 38, wherein said means that generates comprises computer-readable program code means that generates a first triangulation from the initial triangulation by contracting a first edge in the initial triangulation and measuring a first error associated with the first edge contraction; and wherein said means that connects comprises computer-readable program code means that generates a first simplicial homeomorphism for the first triangulation.

40. The product of Claim 39, wherein said means that connects comprises computer-readable program code means that determines a fuzzy rank of a submatrix of a fundamental quadric used by the quadratic error function to measure the first error.

41. A computer program product that models a surface of an object and comprises a computer-readable storage medium having computer-readable program code embodied in said medium, said computer-readable program code comprising:

computer-readable program code means that converts a first triangulation of the surface into a second triangulation of the surface by contracting a first edge in the first triangulation;

computer-readable program code means that determines a fuzzy rank associated with the first edge contraction; and

computer-readable program code means that determines a simplicial homeomorphism based on the fuzzy rank.

42. An apparatus that generates models of objects, comprising:
means for decomposing an initial triangulation of a model into a
quadrangulation of the model defined by a plurality of quadrangular
patches that are joined together at patch boundaries by:

- 5 generating from the initial triangulation of the model a hierarchy
of progressively coarser triangulations of the model using a
sequence of edge contractions to the initial triangulation;
 connecting the triangulations in the hierarchy using
homeomorphisms;
- 10 homeomorphically mapping edges of a coarsest triangulation in
the hierarchy back to the initial triangulation; and
 converting the mapped coarsest triangulation to the
quadrangulation by matching pairs of adjacent triangles in the
coarsest triangulation.

43. The apparatus of Claim 42, wherein said decomposing means
generates a first triangulation from the initial triangulation by contracting a
first edge in the initial triangulation, measuring a first error associated with
the first edge contraction, determining a first fuzzy rank associated with the
first edge contraction and generating a first simplicial homeomorphism
5 based on the first fuzzy rank.

44. The apparatus of Claim 43, wherein said decomposing means
determines an inverse of a composition of the homeomorphisms.

45. An apparatus that models objects, comprising:

means for converting a first triangulation of a surface of an object into a second triangulation of the surface by contracting a first edge in the first triangulation;

5 means for determining a fuzzy rank associated with the first edge contraction; and

means for determining a simplicial homeomorphism based on the fuzzy rank.

46. A method of modeling a surface of an object, comprising the steps of:

generating from an initial triangulation of the surface, a hierarchy of progressively coarser triangulations of the surface by performing a sequence of edge contractions to the initial triangulation using a greedy algorithm that selects edge contractions by their numerical properties;

5 connecting the triangulations in the hierarchy using homeomorphisms; and

homeomorphically mapping edges of a triangulation in the hierarchy back to the initial triangulation.

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47. A method of generating a model of a surface, comprising the steps

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- ~~generating a first triangulated surface;~~

detecting a hole within the first triangulated surface by identifying a plurality of first triangles therein having respective first edges that are not shared by another triangle but collectively define a boundary around an enclosed area devoid of triangles;

filling the hole with a second triangulated surface;

improving quality of the second triangulated surface by:

refining the second triangulated surface into a third triangulated surface having a higher density of triangles therein relative to the second triangulated surface; and

decimating the third triangulated surface into a fourth triangulated surface having a lower density of triangles therein relative to the third triangulated surface using an algorithm that favors generation of equilateral triangles when edges of triangles in the third triangulated surface are contracted, said fourth triangulated surface having fixed vertices on the boundary and floating vertices off the boundary; and

covering the fourth triangulated surface and a portion of the first triangulated surface with a quadrangular NURBS patch;

projecting the floating vertices to the quadrangular NURBS patch; and

generating a fifth triangulated surface comprising the projected/floating vertices and the fixed vertices.

48. A method of generating a model of a surface, comprising the steps of:

generating a first triangulated surface;

5 detecting a hole within the first triangulated surface by identifying a plurality of first triangles therein having respective first edges that are not shared by another triangle but collectively define a boundary around an enclosed area devoid of triangles;

filling the hole with a second triangulated surface;

improving quality of the second triangulated surface by:

10 refining the second triangulated surface into a third triangulated surface having a higher density of triangles therein relative to the second triangulated surface; and

decimating the third triangulated surface into a fourth triangulated surface having a lower density of triangles therein relative to the third triangulated surface, using an algorithm that favors generation of equilateral triangles when edges of triangles in the third triangulated surface are contracted; and

generating a fifth triangulated surface from the fourth triangulated surface using an energy minimization operation that evaluates a local measurement of shape selected from the group consisting of angles, ^{linearized therefor} ~~linear~~ expressions between adjacent triangles, angles around vertices and ~~linear~~ expressions of curvature.

49. The method of Claim 12, wherein said converting step is followed by the step of:

reconfiguring the quadrangulation by performing a plurality of local transformations on the quadrangulation so that a deficiency associated with the reconfigured quadrangulation is less than a deficiency associated with the quadrangulation.

50. A method of modeling a surface of an object, comprising the steps

of:

converting a first triangulation of the surface into a first quadrangulation of the surface, by:

5 converting the first triangulation of the surface into a second triangulation of the surface by contracting a first edge in the first triangulation;

determining a fuzzy rank associated with the first edge contraction; and

10 determining a simplicial homeomorphism based on the fuzzy rank; and

reconfiguring the first quadrangulation of the surface into a second quadrangulation of the surface by performing a plurality of local transformations on quadrangles in the first quadrangulation so that a potential $\Phi_2(Q)$ associated with the second quadrangulation is less than a potential $\Phi_1(Q)$ associated with the first quadrangulation.

51. The method of Claim 50, where $\Phi_n(Q) = K\Delta_n(Q) + F_n(Q)$ and $K \geq 2$, $\Delta_n(Q)$ represents a deficiency of a respective n th quadrangulation and $F_n(Q)$ represents the number of quadrangles in the respective n th quadrangulation.